图像认知中的优化问题

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清华大学电子工程系青年教师论坛

二零一一年五月

- 优化问题
- 总变差方法 (Total Variation Methods)
- 主动形状模型 (Active Shape Model)
- 图割图像分割 (Graph Cut Image Segmentation)
- 人脸图像光照变换
- 焊缝检测
- 多生物特征融合

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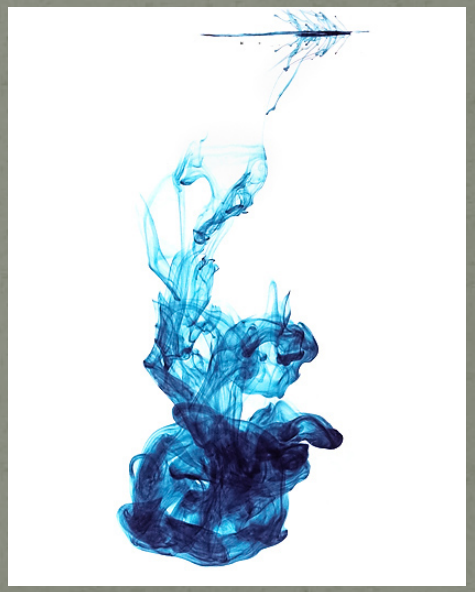
优化问题

自然过程往往最小化（最大化）某个物理量

势能最小



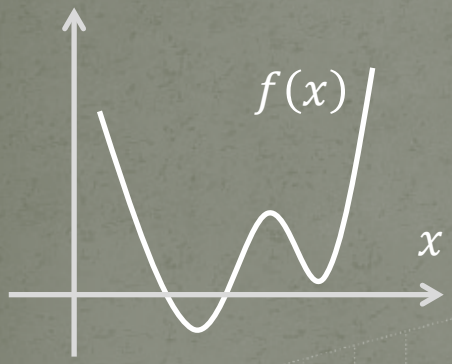
熵最大

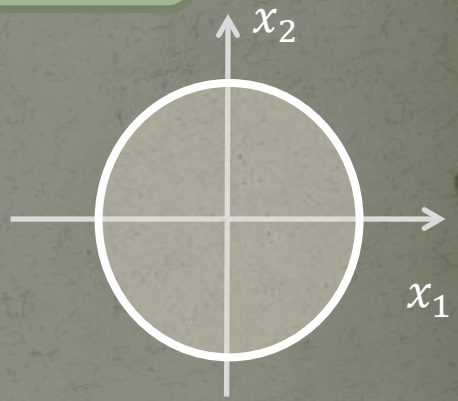
优化问题

目标函数
Objective Function

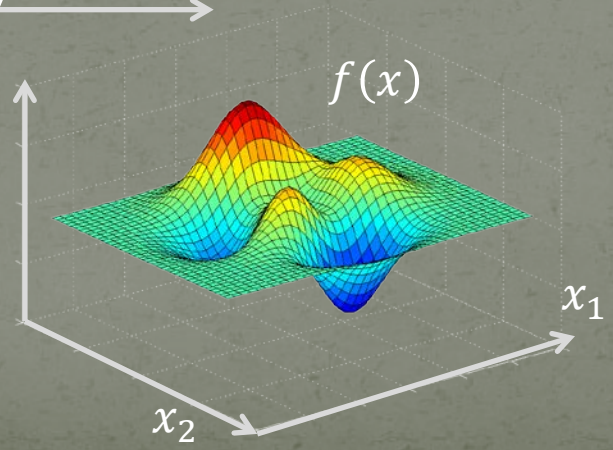
约束条件 $g_i(x) = 0$
Constraints $h_i(x) \leq 0$

$$minimize \ f(x) \quad subject\ to \quad x \in C, C \subseteq \mathbf{R}^n$$





$x_1^2 + x_2^2 \leq 1$



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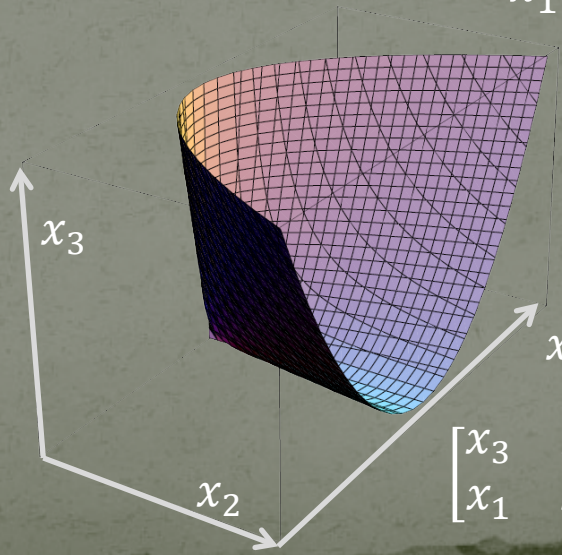


$$\begin{bmatrix} x_3 & x_1 \\ x_1 & x_2 \end{bmatrix} \in S_+^2$$

优化问题

然而，’一般性’的优化问题是不可解的！[Nesterov ‘04]

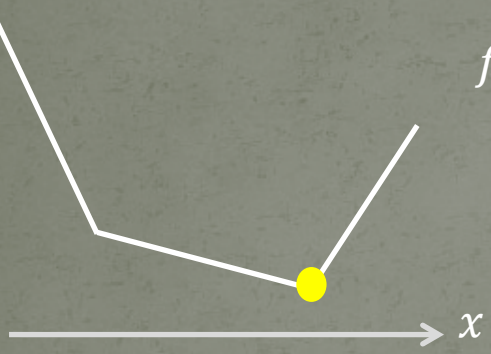
$minimize\ f(x) \quad subject\ to\ \{x \in \mathbf{R}^n : 0 \le x_i \le 1, i = 1, \dots, n\}$

Lipschitz条件： $|f(x) - f(y)| \le L|x - y|_\infty, \forall x, y \in C$

设L=2、n=10，求误差不超过ϵ=1%的最优解的算法复杂度为

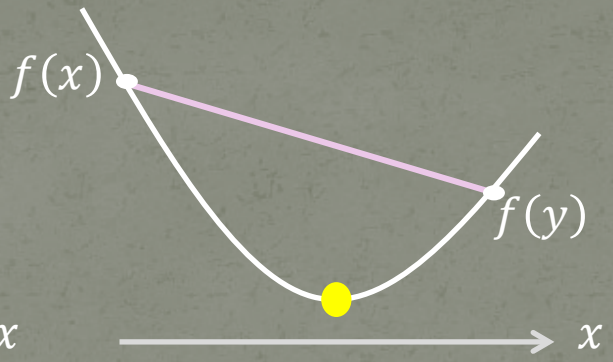
$\left\lfloor \frac{L}{2\epsilon} \right\rfloor^n = 10^{20}$，运行时间 > 10000年！？

优化问题



线性优化

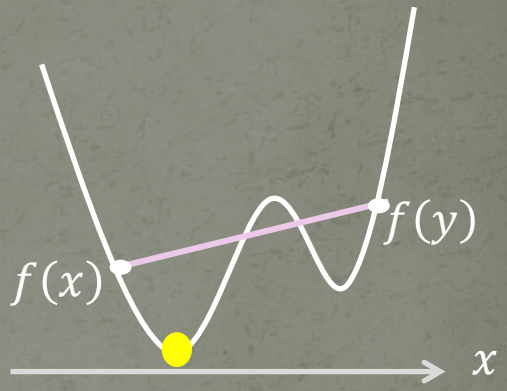
1984年以前...



凸优化

1984年，Karmarkar突破性地提出了多项式时间的Interior Point优化算法，‘解决’了凸优化问题。

[Karmarkar'84, SCI: 1314]



非凸优化

尚无稳定、有效的优化算法 ...

优化问题

"The great watershed in optimization isn't between linearity and non-linearity, but convexity and non-convexity"

[Rockafellar '93]

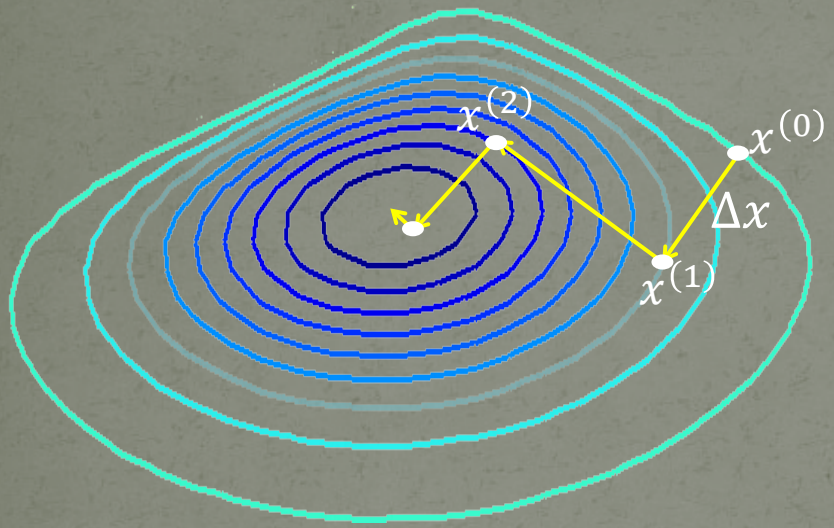
凸优化

- 保证可以找到全局最优解
- 优化算法稳定高效
- 优化结果与初始值无关
- 对问题的建模较不精确

非凸优化

- 基本无法找到全局最优解
- 优化算法因人而异
- 优化结果依赖初始值
- 对问题的建模精确

优化问题



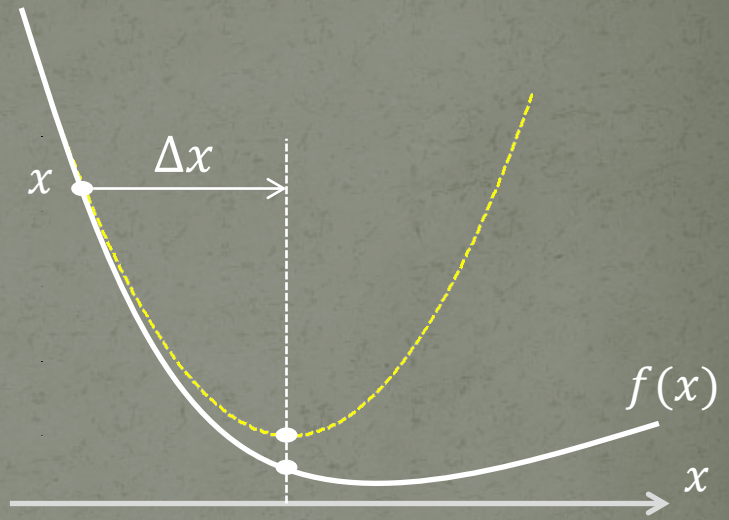
梯度下降法

$$\Delta x = -\nabla f(x)$$

最陡下降法

$$\Delta x = argmin\{\nabla f(x)^T v \mid \|v\| \leq 1\}$$

凸优化基本算法



牛顿下山法

$$\Delta x = -\nabla^2 f(x)^{-1} \nabla f(x)$$

…… ……

优化问题

Interior Point Method

$$minimize\ f(x) \quad subject\ to \quad h_i(x) \leq 0$$

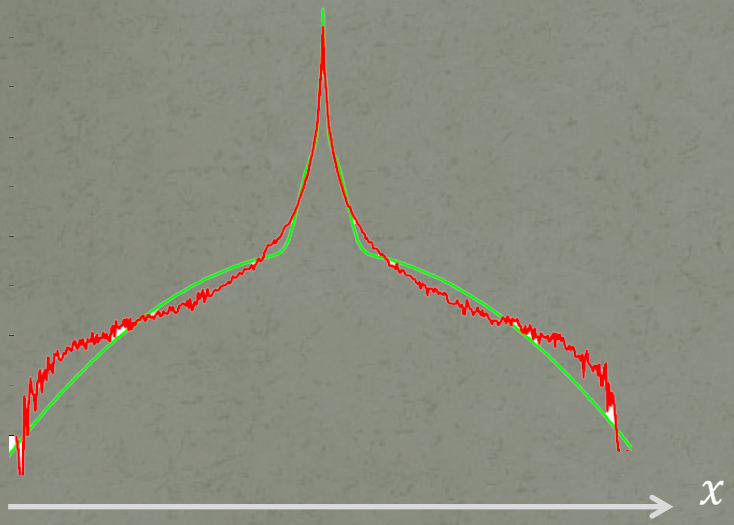
$$minimize\ f(x) - \lambda \sum_i log(-h_i(x))$$

32 iterations is all that you need ...

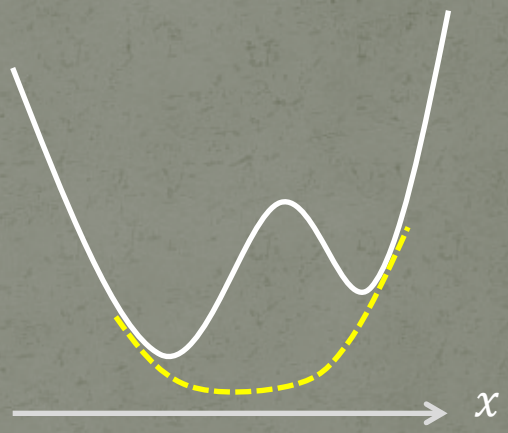
优化问题

非凸优化

凸优化算法 + 启发式搜索 + 运气



分段凸逼近
[Fergus'06]



局部凸松弛
[Kahl'08]

优化问题

大多数图像认知问题都可转化为优化问题 ...

minimize ??
subject to ? ? ?

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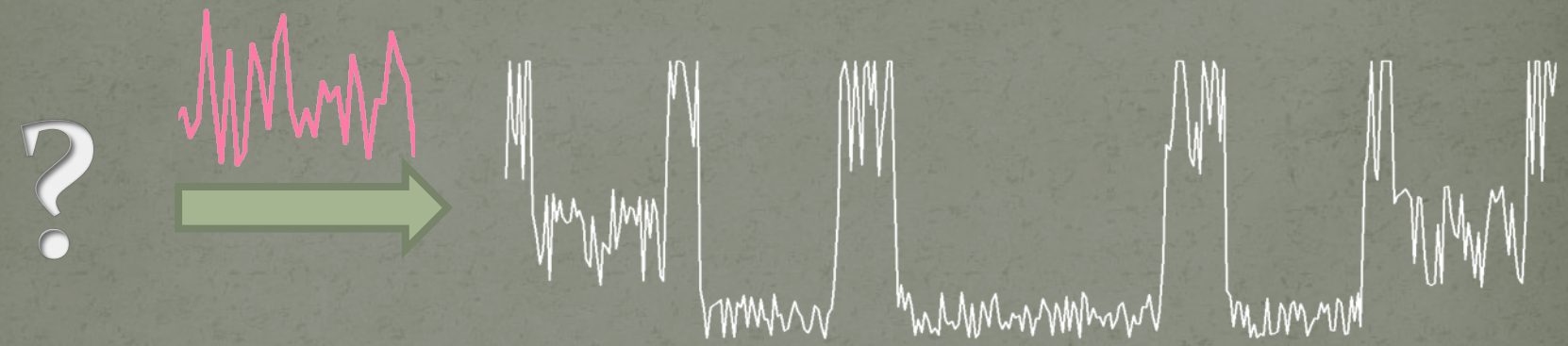
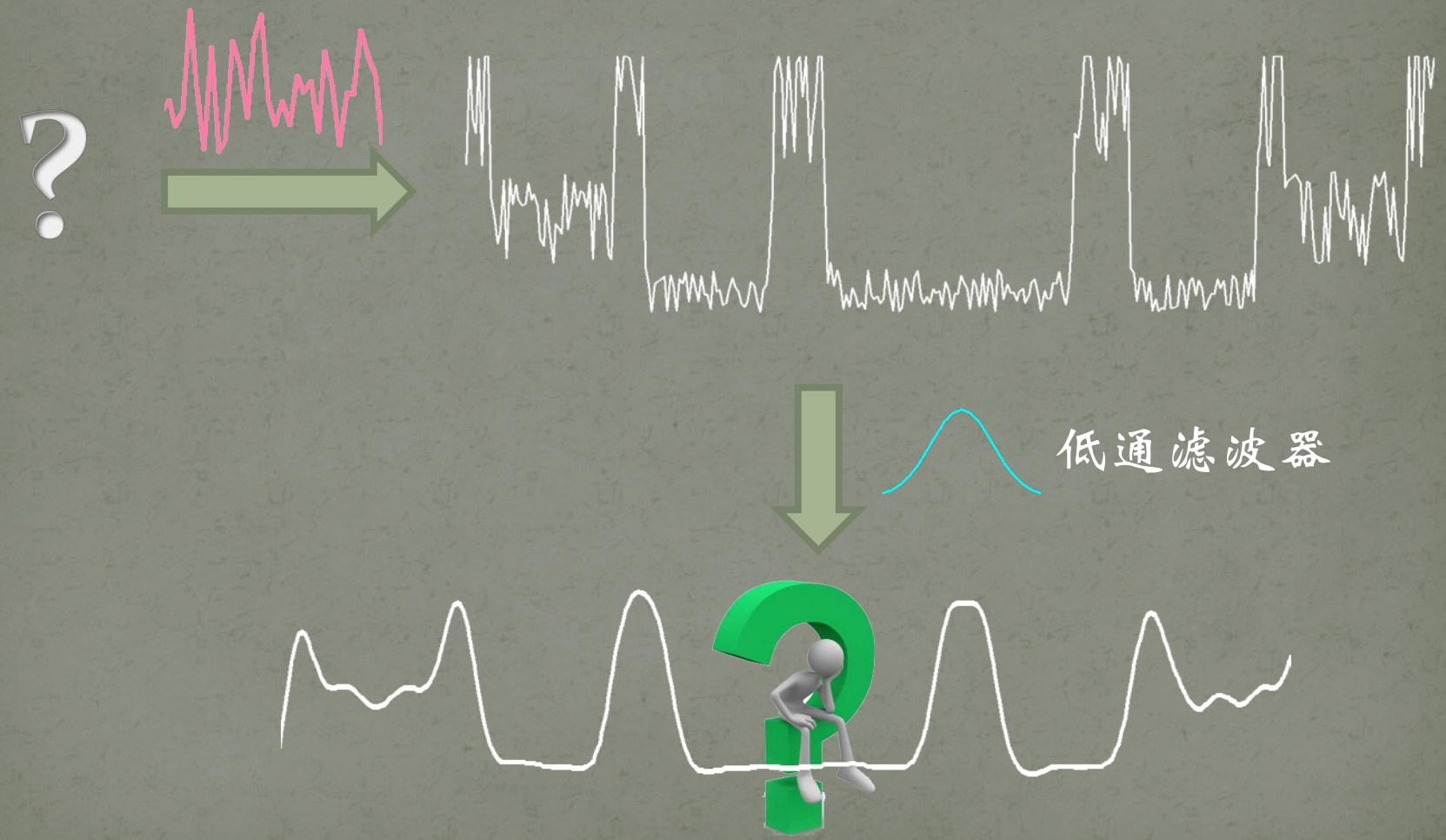
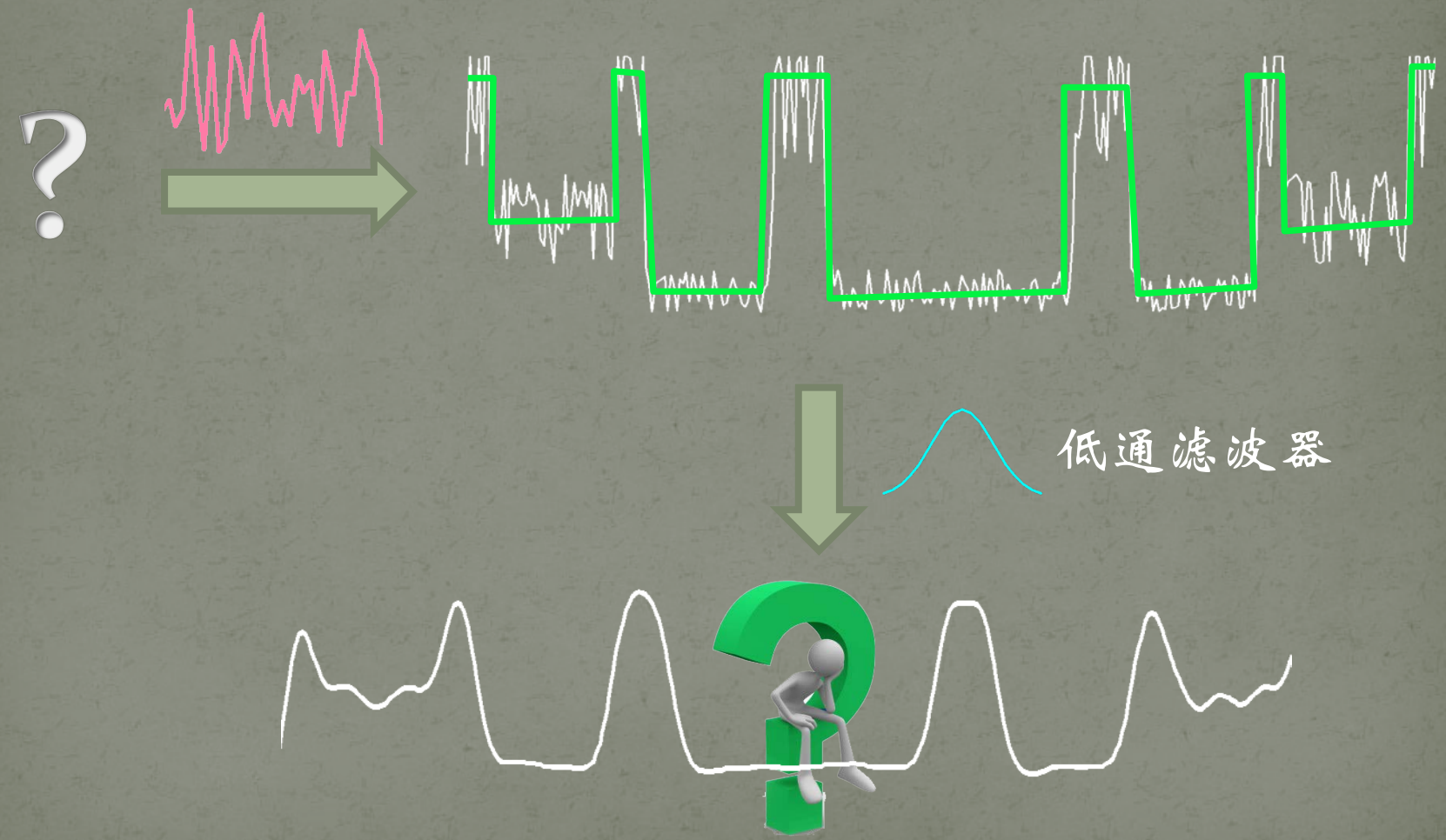
总变差方法



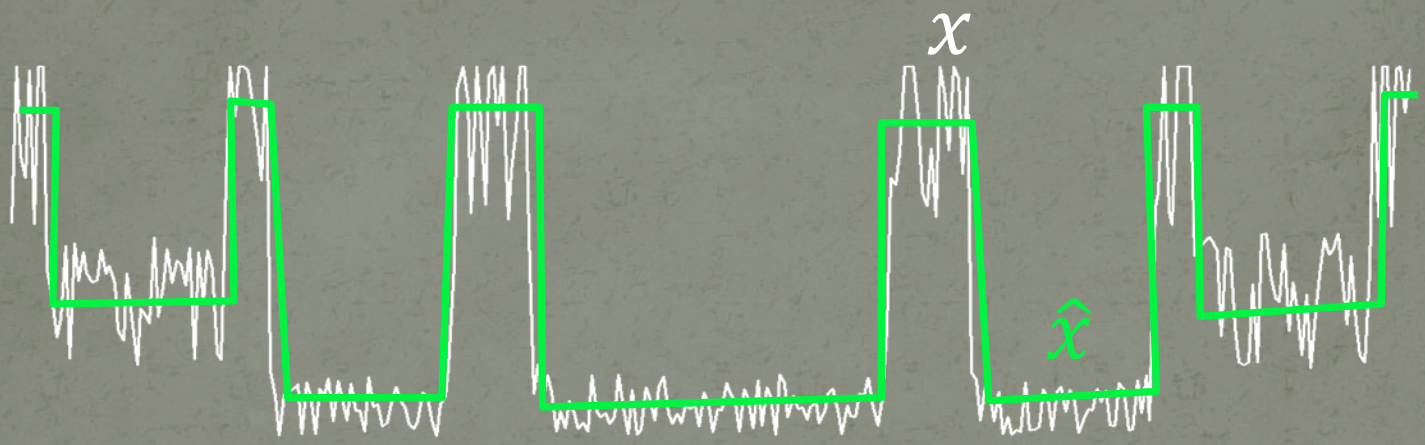
总变差方法



总变差方法



总变差方法



认知一：$\hat{x}$和x非常接近：$\|\hat{x}-x\|^2$ ↓

认知二：$\hat{x}$大多数区域平坦：$\int |\nabla \hat{x}| d\hat{x}$ ↓

Total Variation

总变差方法

$$minimize \quad \frac{1}{2}\|\hat{x} - x\|^2 + \int |\nabla \hat{x}| d\hat{x}$$

离散形式： $$minimize \quad \frac{1}{2}\sum_i \|\hat{x}_i - x_i\|^2 + \sum_i |\hat{x}_{i+1} - \hat{x}_i|$$

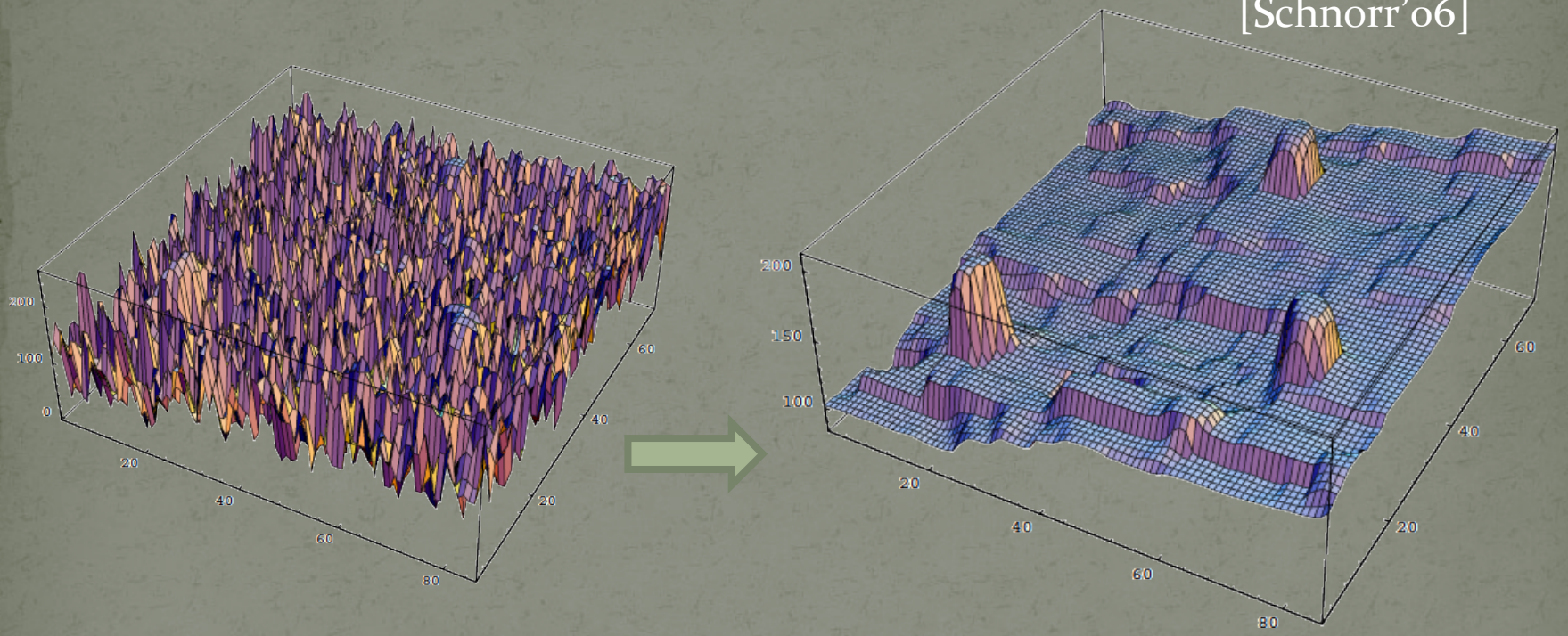
It is convex!

总变差方法

总变差图像去噪
[Rudin'92, SCI: 1802]

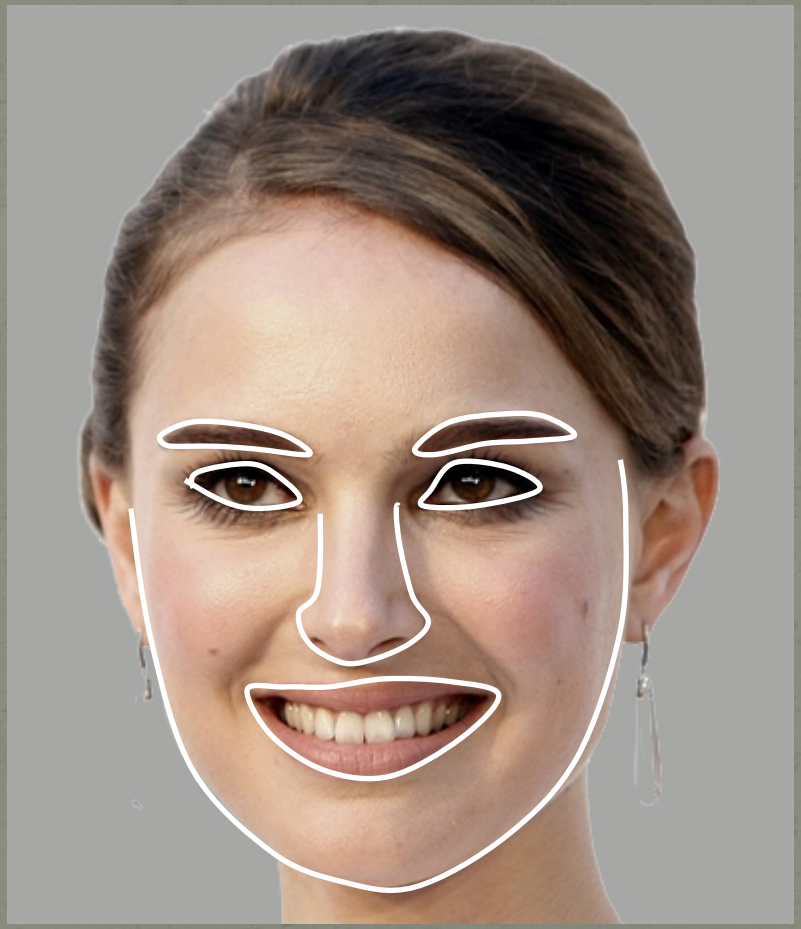
总变差方法

[Schnorr'06]



目前，总变差方法被广泛应用于图像去噪、图像重建、图像分割等问题中。

主动形状模型



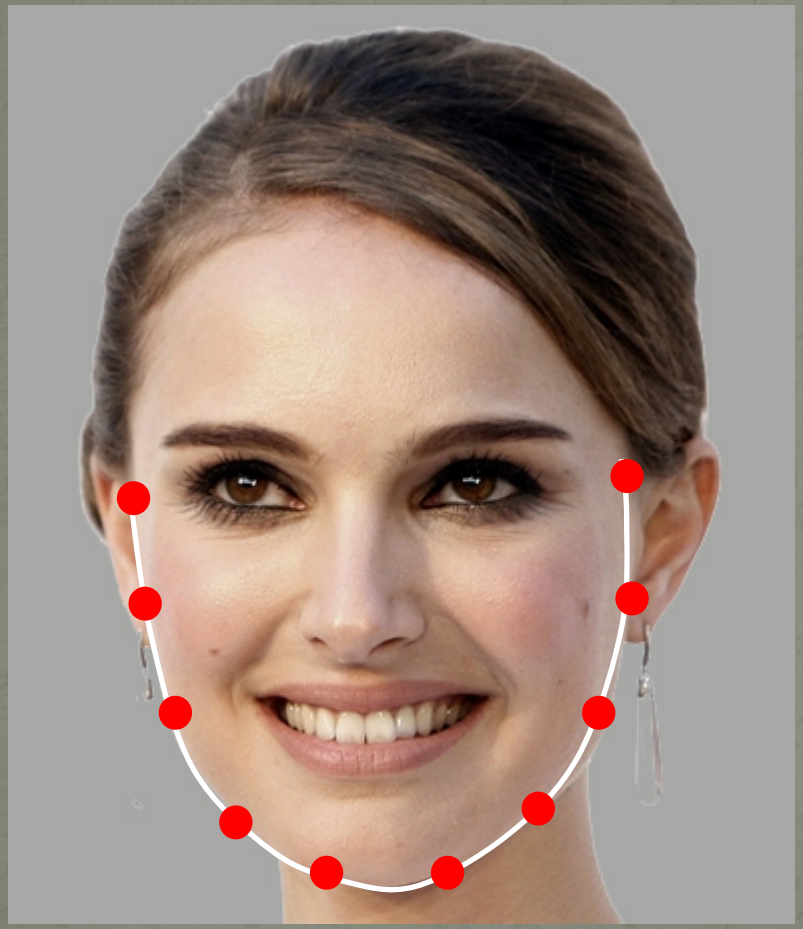
勾勒出人脸及五官形状?

主动形状模型

自动勾勒形状

边缘检测？

主动形状模型

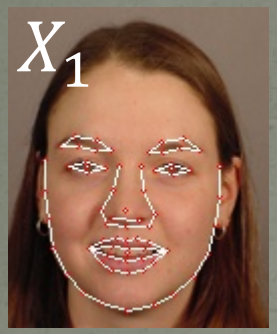


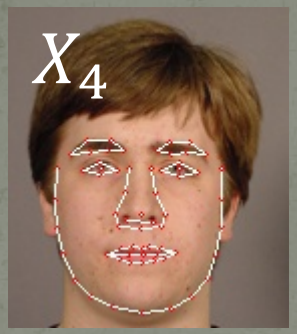
认知一：人脸形状大体上相似（形状模型）。

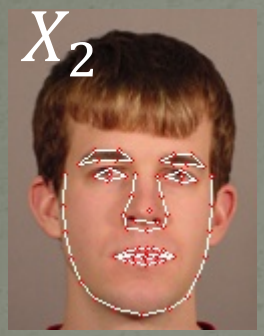
用一个点序列表示形状。

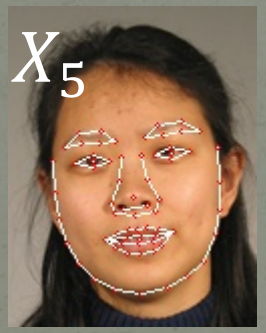
$$X = [x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n]^T$$

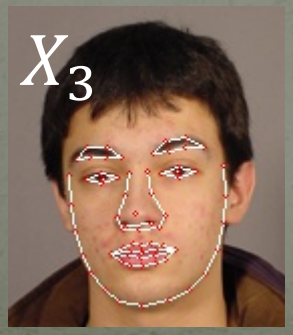
主动形状模型











……

形状模型

在一系列（几百）人脸图像上手工标注形状，构成形状训练集合：$X_1, X_2, \ldots, X_m$

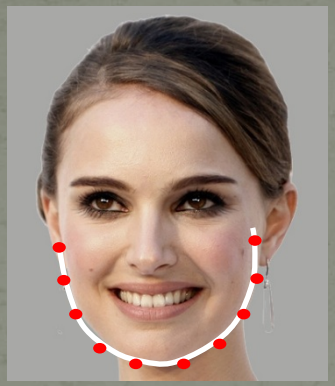
以协方差矩阵特征向量表示

$$X = \text{均值}(X_i) + b \cdot \text{方差}(X_i)$$

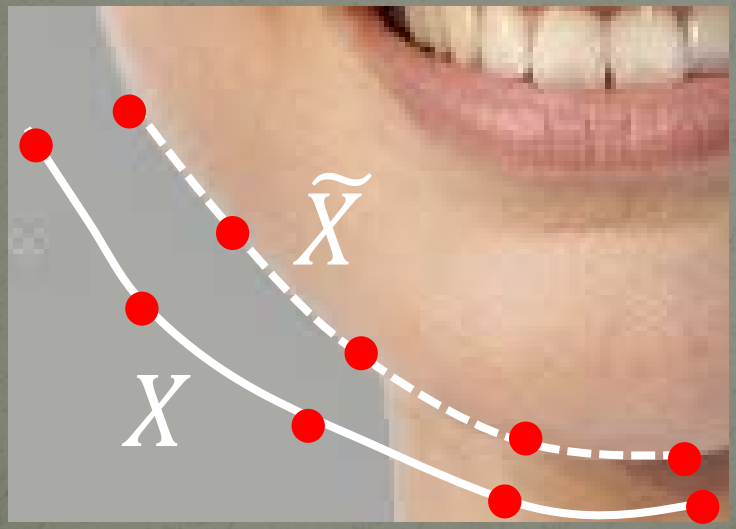
模型参数

$|b_i| \leq 3\sqrt{\lambda_i}$

主动形状模型



认知二：形状点应在人脸图像的边缘附近。



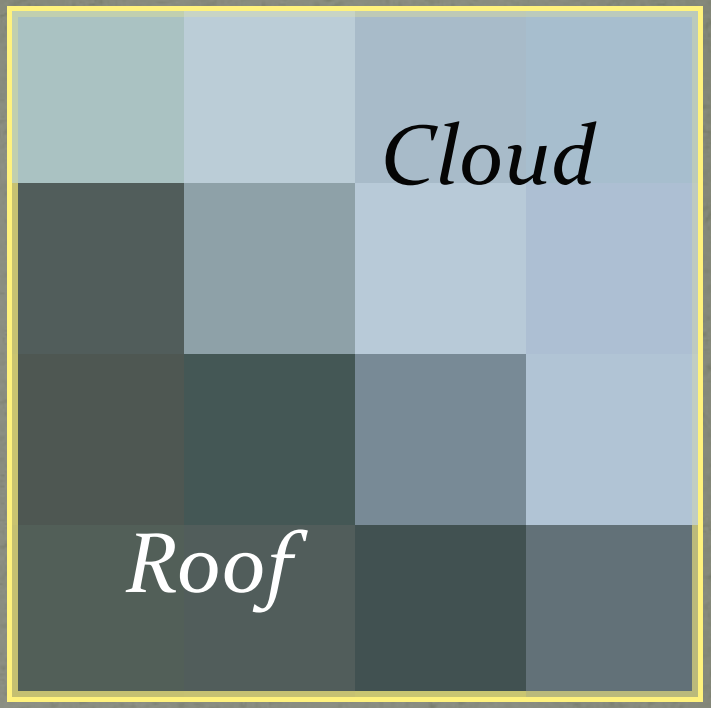
$$minimize \ \left\|X(b) - \tilde{X}\right\|^2$$
$$subject\ to\ |b_i| \leq 3\sqrt{\lambda_i}$$

[Cootes'95, SCI: 1831]

図割図像分割

図像分割

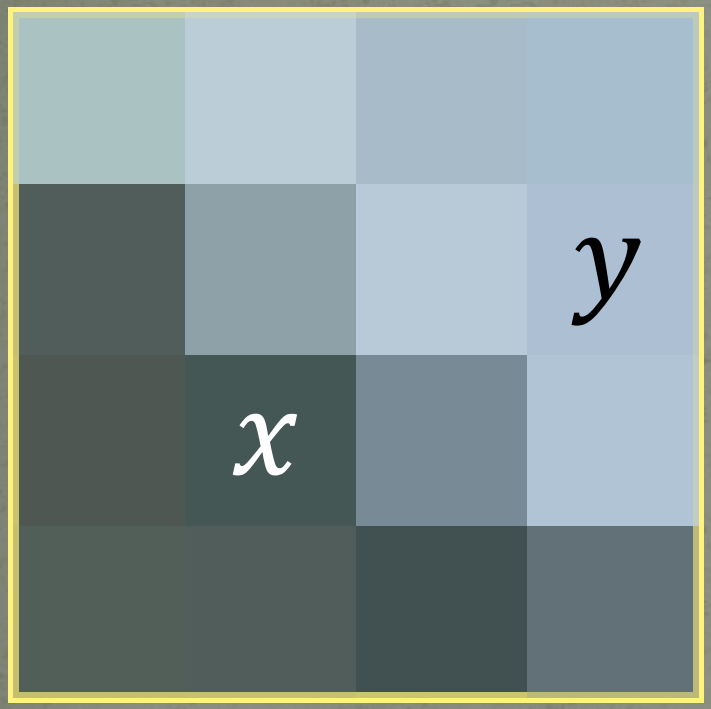
图割图像分割



认知一：颜色(亮度)接近的像素属同一物体

认知二：空间位置相近的像素属同一物体

图割图像分割



定义任意两像素间的关联度

- 颜色关联度 (Affinity)

$$aff_C = exp(-\|c(x) - c(y)\|/2\sigma_c^2)$$

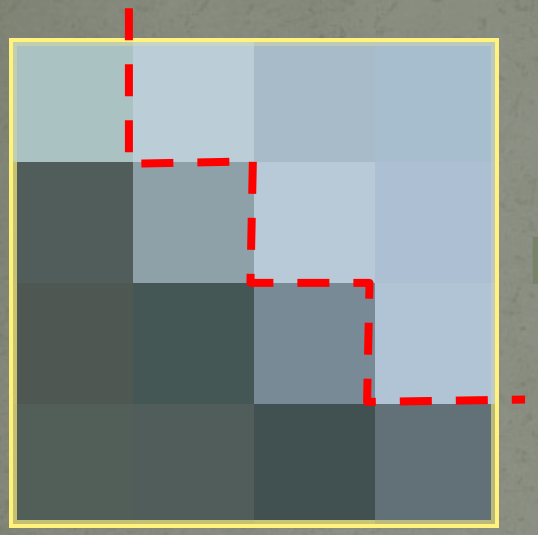
- 距离关联度

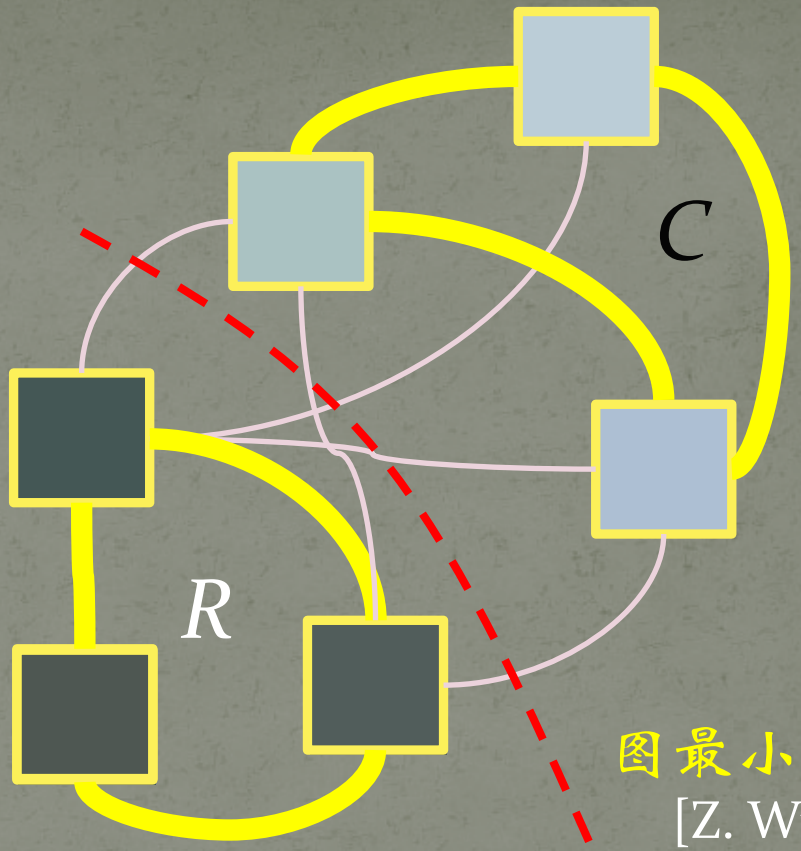
$$aff_D = exp(-\|x - y\|/2\sigma_D^2)$$

- 总关联度

$$aff(x, y) = aff_C + \lambda * aff_D$$

图割图像分割





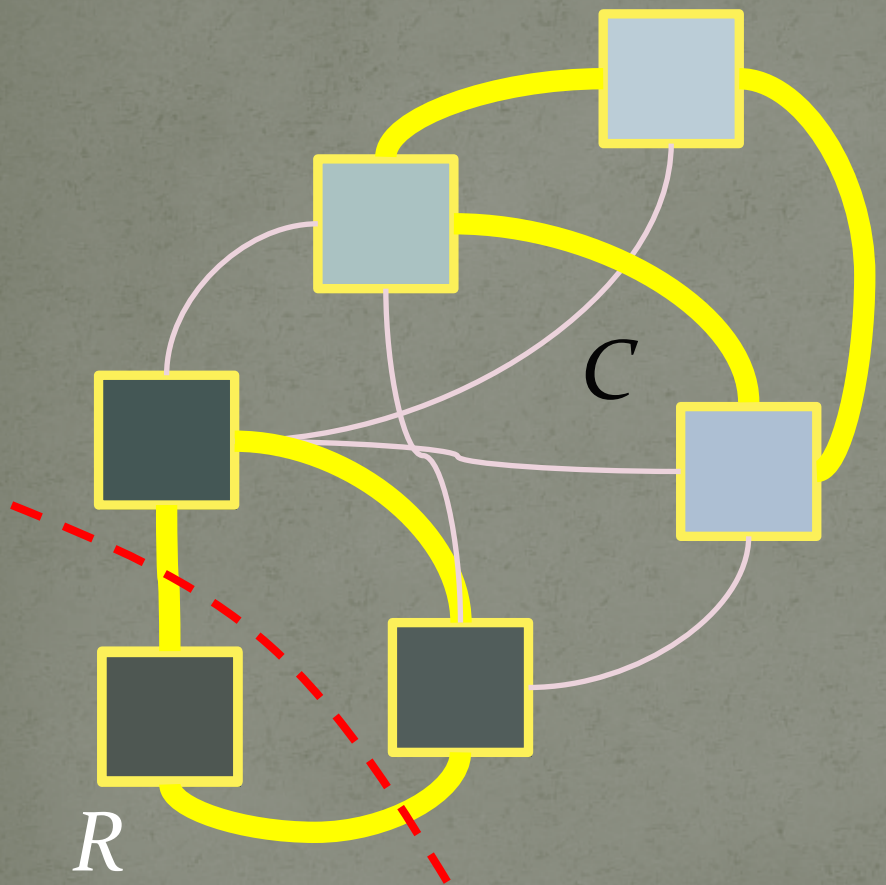
构造一个图(Graph)V：
每个像素为顶点，两像素间路径权重(ω)等于两像素间的关联度。

图最小割问题
[Z. Wu '93]

$$minimize\ \ cut(R,C) = \sum_{\substack{x\in R \\ y\in C}} \omega(x,y)$$

多项式时间算法 [Ford & Fulkerson' 62]

图割图像分割



倾向于割到孤立顶点!

Normalized Cuts:

$$Ncut(R,C) = \frac{cut(R,C)}{aff(R,V)} + \frac{cut(R,C)}{aff(C,V)}$$

[J. Shi '00, SCI: 1488]

More Examples …

- Low-rank matrix approximation using L1 norm [Eriksson'10, CVPR Best Paper]
- Non-rigid surface detection [Pilet'05, CVPR Best Paper]
- Graph cut based inference with co-occurrence statistics [Ladicky'10, ECCV Best Paper]
- LMI relaxation in geometric reconstruction [Kahl'05, ICCV Best Paper]
- Compressive Sensing (L1 Optimization)
- Local Linear Embedding (Quadratic Optimization)
- Support Vector Machine (Quadratic Optimization)
- ……

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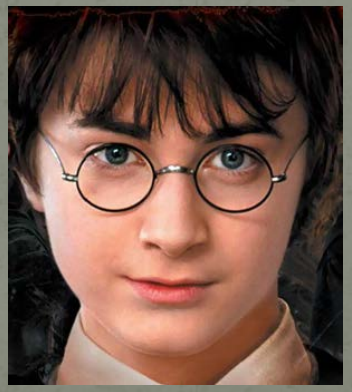
人脸图像光照变换



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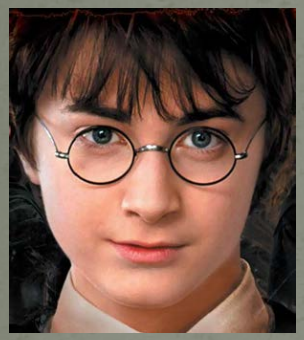
人脸图像光照变换

比值图像 R

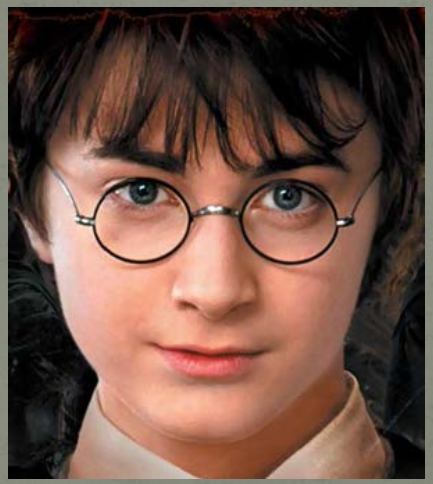


人脸图像光照变换

认知一：图像总体的亮度变化由比值图像决定

认知二：局部的亮度变化平缓

$$minimize \sum_i \left(\sum_{j \in \omega_i} \left(\hat{I}(j) - \alpha_i I(j) \right)^2 + \lambda(\alpha_i - R_i)^2 \right)$$ [Chen'10]



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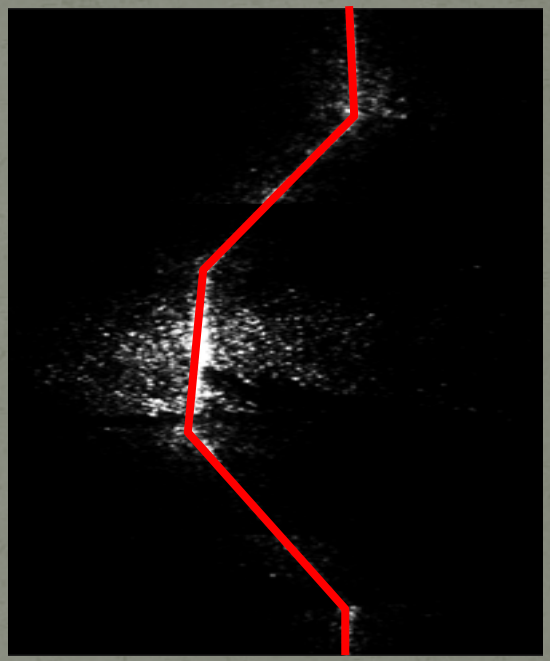
$\hat{I}$

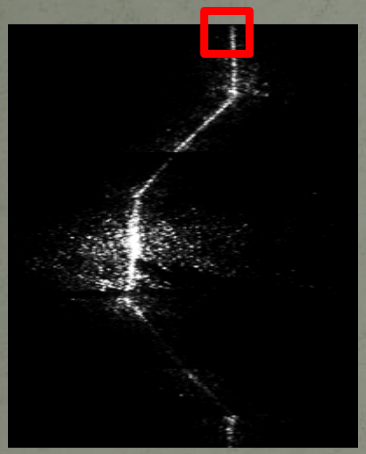
焊缝检测



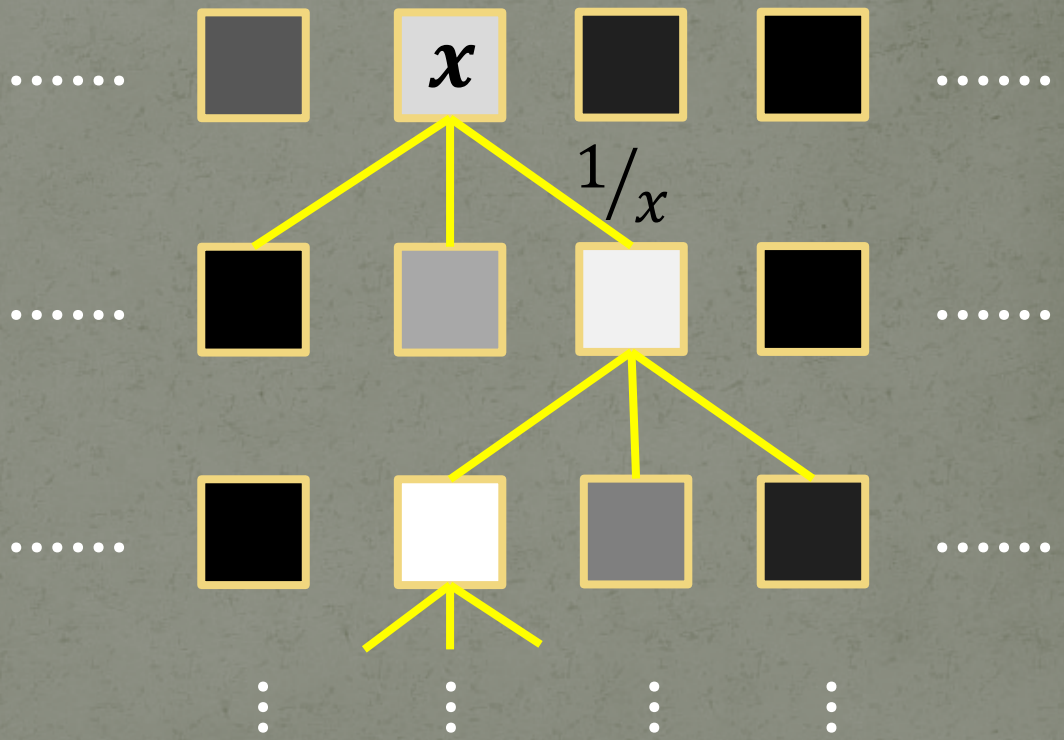
认知一：连续的单像素构成的路径

认知二：尽量通过高亮的区域

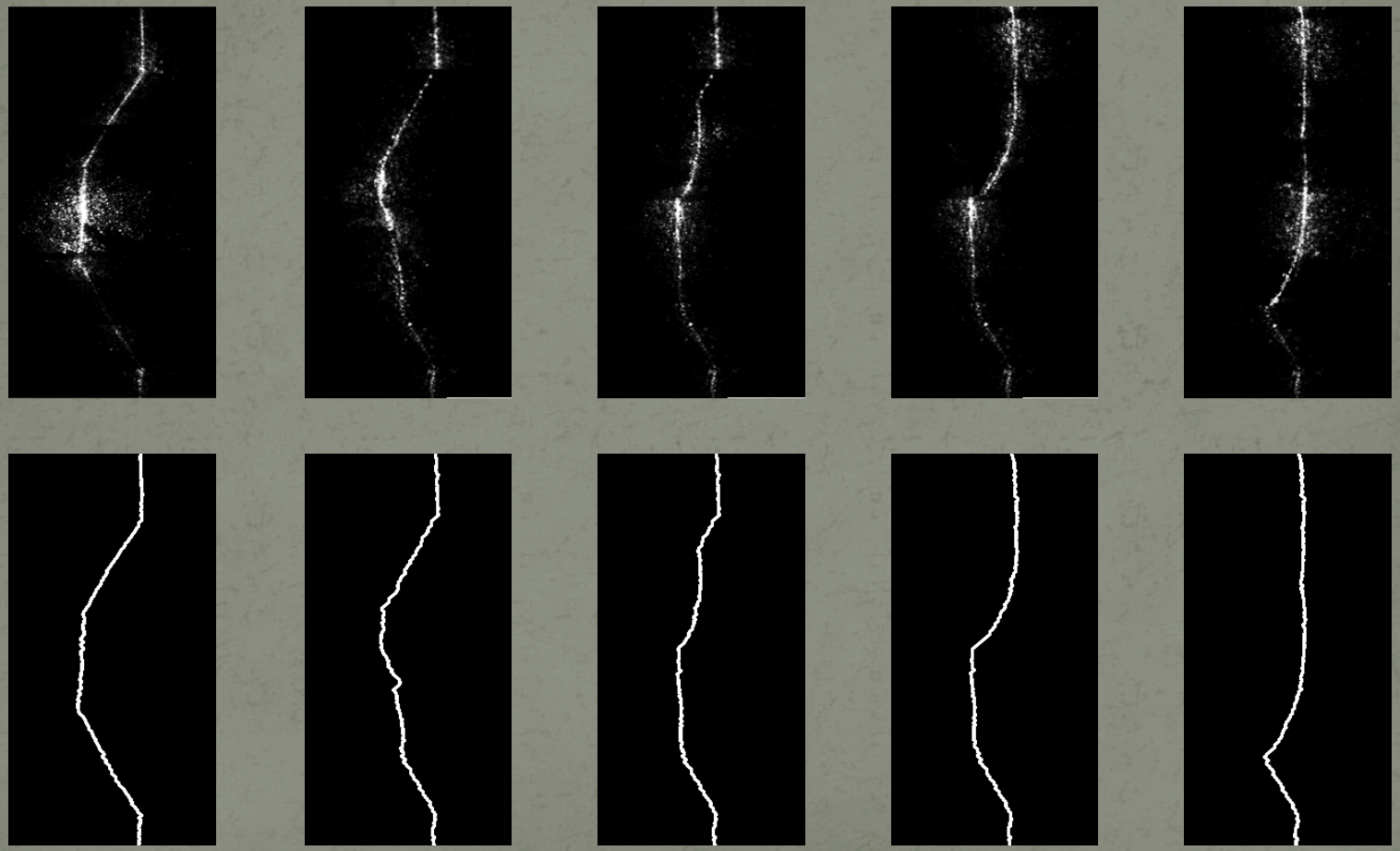
焊缝检测



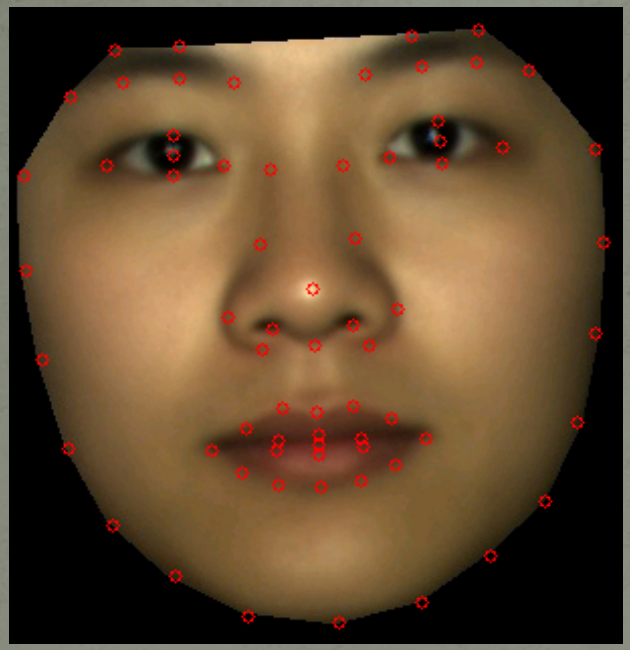
图最短路径 [Dijkstra'59]



焊缝检测



多生物特征融合

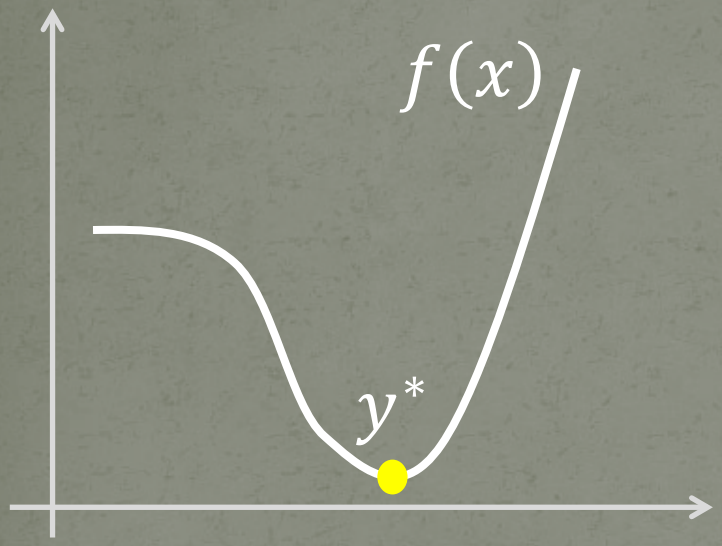


设定局部特征的权重提高识别率？

$$minimize \quad \sigma_G/\left(\mu_G - \Psi\left(I^T * \omega, r\right)/r\right)$$

$$subject\ to \quad \sum_{i=1}^{N} \omega_i = 1$$

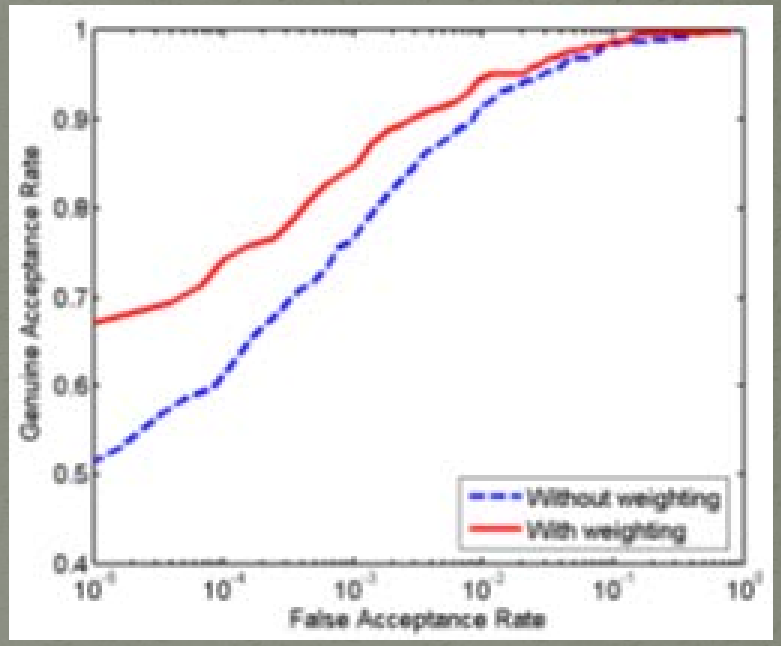
多生物特征融合



拟凸优化二分算法

- given $y_l \leq y^*, y_u \geq y^*$
- do
- $t = (x_l + x_u)/2$
- $minimize\ 0 \quad s.t.\ f(x) \leq t$
- if succeed $y_u = t$
- else $y_l = t$
- while $y_u - y_l > \varepsilon$

多生物特征融合


Genuine Acceptance Rate
False Acceptance Rate
Without weighting
With weighting

Everything is not optimization …


Thanks

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